limiting box chord and box aspect ratio. On the other hand, it will permit paneling of delta wings with little effort and without violating the guidelines. A limit of one-quarter of a wavelength on both the spanwise and chordwise extent of a doublet line is suggested to ensure reasonably linear convergence behavior with respect to simultaneous refinement of spanwise and chordwise paneling.

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Rapid Prediction of Wing Rock for Slender Delta-Wing Configurations

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Introduction

SINCE the initial experiments by Nguyen et al.[1] and Levin and Katz,[2] which showed that an 80-deg delta wing at high angles of attack exhibits the limit-cycle oscillations in roll, referred to as slender wing rock, Arena and Nelson[3,5–7] and Arena et al.[4] have been performing a series of tests and analyses of a sharp-edged 80-deg delta wing, aimed to provide insight into the fluid mechanical processes causing slender wing rock. In Fig. 1 theirexperimental wing rock results[7] are compared to those in Refs. 1 and 2. How the difference between the results in Refs. 1 and 2 was caused by the different experimental setups is discussed in Ref. 8. The likely reason that the test results in Ref. 7 show a higher maximum limit-cycle amplitude than those in Refs. 1 and 2 (Fig. 1) is that the damping provided by the bearing friction in Refs. 1 and 2 was almost completely eliminated in Ref. 7 by the use of an air bearing. The predicted start of wing rock[9] (Fig. 2) agrees well with these friction-free test results (Fig. 1). In addition to knowing when slender wing rock will start for a particular leading-edge sweep[9] (Fig. 2), the vehicle designer also needs to know how large the limit-cycle amplitude can become when the critical angle of attack is exceeded; e.g., $\alpha > 20$ deg for $\Lambda = 80$ deg (Fig. 2). A simple analysis for the determination of the maximum possible limit-cycle amplitude is described, which produces results that compare well with the experimental results in Fig. 1 for the 80-deg delta wing.

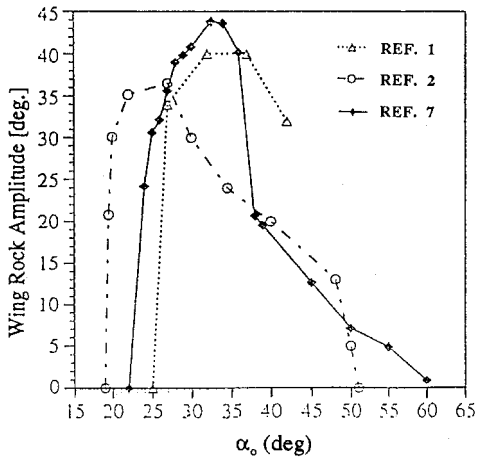

Fig. 1 Experimentally determined wing-rock-amplitude envelope for an 80-deg delta wing.

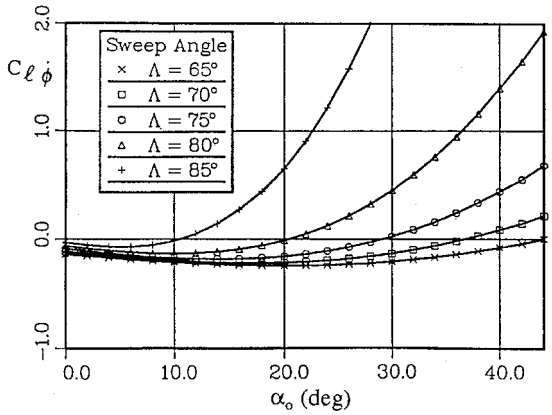

Fig. 2 Effect of leading-edge sweep on delta-wing roll damping at high angles of attack.[9]

Maximum Limit-Cycle Amplitude

Experimental results for the 80-deg delta wing[4] (Fig. 3a) show that when the roll angle is increased from $\phi = 0$ to the maximum wing rock value, $\phi = 44$ deg (Fig. 1), the left, laterally leeside vortex increases its distance above the wing surface from $\zeta_v = z_v/s \approx 0.47$ to $\zeta_v \approx 1.04$ while moving outboard from $\eta_v = y_v/s \approx -0.65$ to $\eta_v \approx -1.70$. Combining the ζ_v, η_v results in Fig. 3a gives the vortex location relative to the wing as a function of roll angle (Fig. 3b). The combined effects of the vortex-induced suction and downwash will contribute

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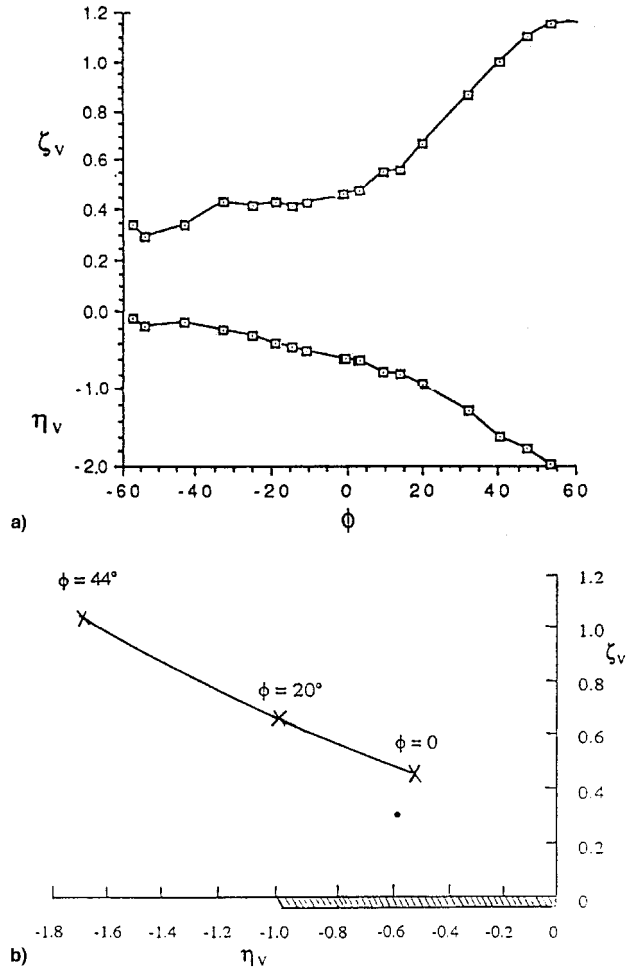

Fig. 3 Effect of roll angle on vortex position above the left, leeward side of 80-deg delta wing at α = 30 deg[4]: a) measured $\eta_v(\phi)$ and $\zeta_v(\phi)$, and b) vortex position defined by $\eta_v(\phi)$ and $\zeta_v(\phi)$.

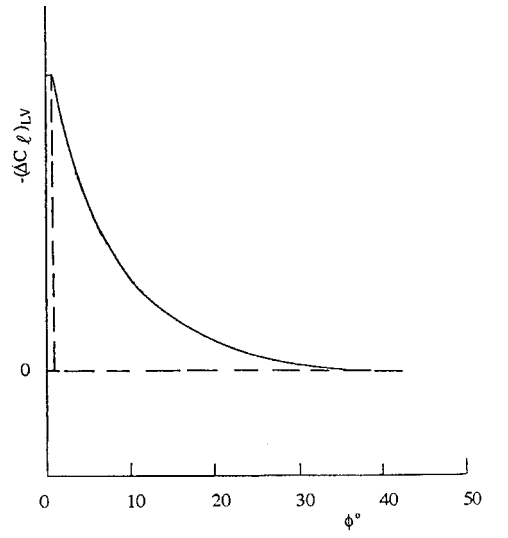

Fig. 4 Conceptual contribution to the rolling moment by the left, leeward leading-edge vortex.

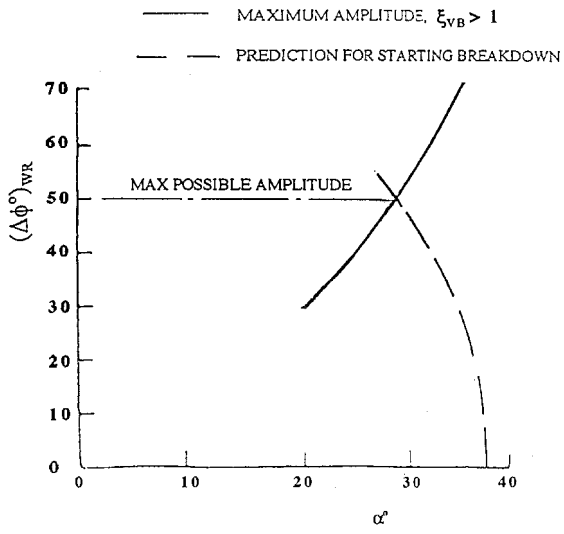

Fig. 5 Predicted maximum possible wing-rock amplitude for 80-deg delta wing.

negligibly to the rolling moment on the left wing half at ϕ = 44 deg compared with ϕ = 20 and 0 deg. The loss of vortex-induced lift on the left, leeward wing half with increasing ϕ generates a negative, statically stabilizing contribution $(\Delta C_l)_{LV}$ to the rolling moment, which through the time lag effect becomes dynamically destabilizing.[10] The conceptual variation of the loss of vortex-induced rolling moment, $-(\Delta C_l)_{LV} = f(\phi)$, is shown by a solid line in Fig. 4. The dashed line represents the approximation of the vortex liftoff characteristics that was used in Ref. 11 to determine the maximum possible wing-rock amplitude. Earlier analysis of limit-cycle oscillations in pitch of the Polaris re-entry body[12] provided convincing evidence that such an approximation would only slightly overestimate the maximum possible limit-cycle amplitude.

Based on the experience with the Polaris re-entry body,[12] an integrated measure $C_{l\dot{\phi}}$ of the nonlinear roll damping was derived in Ref. 11, using the dashed-line approximation in Fig. 4. The limit-cycle amplitude $\Delta\phi_{WR}$ of the roll oscillations is defined by $C_{l\dot{\phi}} = 0$. This computed wing-rock amplitude was proportional to the vortex-induced normal force C_{NV} lost on the leeward wing half through the vortex liftoff. As this vortex-induced force C_{NV} is proportional to $\sin^2\alpha$ (Ref. 13), the predicted maximum wing-rock amplitude increases very quickly with α (solid line in Fig. 5). Also shown by the dashed line in Fig. 5 is the angle of attack, $\alpha_{VB} = \tan^{-1}(\tan\alpha_0 \cos\Delta\phi_{WR})$, where vortex breakdown will start to occur on the windward half of the delta wing when accounting for the effect of $\Delta\phi_{WR}$ on the effective leading-edge sweep (α_0 is the inclination of the roll axis). In Ref. 10 it was demonstrated that the occurrence of vortex breakdown on the windward wing half generated roll damping. Furthermore, it was argued that the fast movement of vortex breakdown forward of the trailing edge[14] (Fig. 6) would, on the slender delta wings prone to wing rock ($\Lambda > 70$ deg in Fig. 6), provide a sudden, large increase of the roll damping that would stop any further growth of the wing-rock amplitude. Figure 5 shows that the intersection of the two curves, defining the maximum possible magnitude of the wing-rock amplitude, exceeds the maximum amplitudes measured on the 80-deg delta wing[1,2,7] (Fig. 1).

The $\alpha_0 - \theta_A$ boundary for the occurrence of slender wing rock, defined by Ref. 9 and Fig. 2, is shown in Fig. 7. The dashed portions indicate that wing rock is likely to be prevented from occurring; at $\theta_A > 18$ deg by the burst-induced damping[11,14] (Fig. 6), and at $\theta_A \leq 7$ deg by the presence of asymmetric vortex burst[15] and the roll damping generated by the associated roll trim $|\phi_0| > 0$. It is shown in Ref. 16 that for the 80-deg delta wing, $|\phi_0| = 10$ deg would generate an equivalent sideslip angle $|\beta| = 5$ deg at α = 30 deg. As it is likely that $|\phi_0|$ will exceed 10 deg and the experimental results[1] show that $|\beta| > 5$ deg provides roll damping for $\alpha < 35$ deg, wing rock will not occur for $\theta_A \leq 7$ deg.

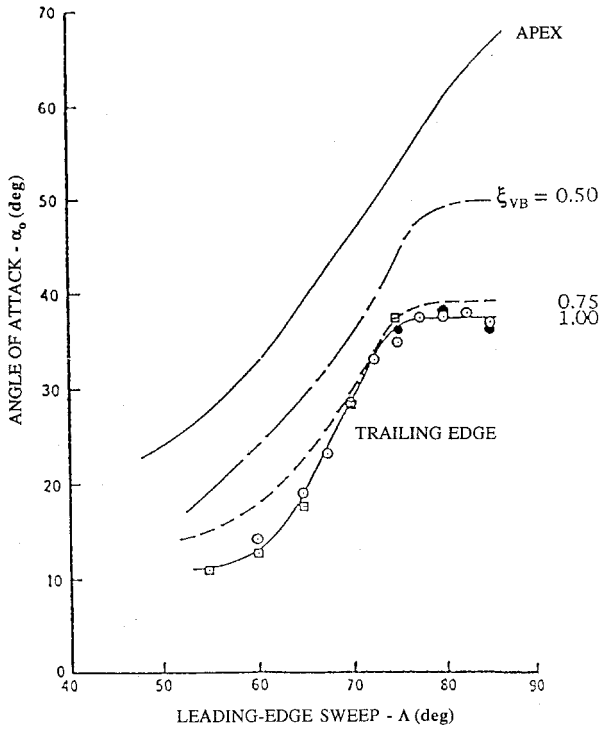

Fig. 6 Vortex breakdown position on sharp-edged delta wings.[14]

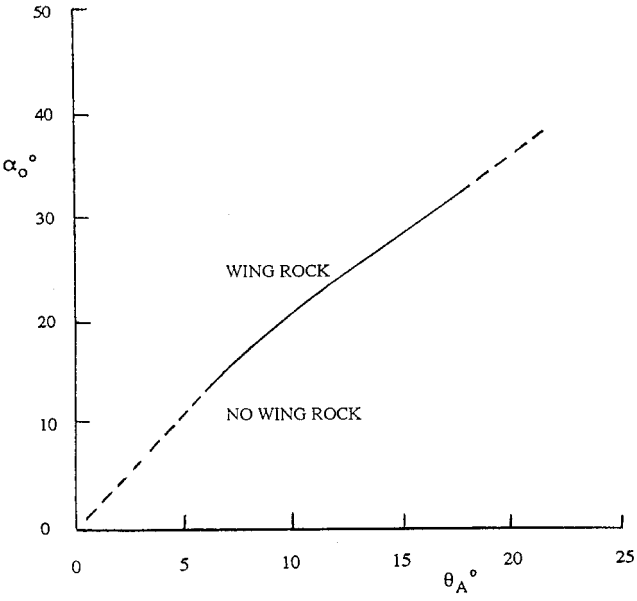

Fig. 7 Predicted wing-rock boundary for slender delta wings.

Conclusions

The information that the vehicle designer needs early in the design cycle is 1) the angle-of-attack/leading-edge-sweep range in which slender wing rock will occur and 2) the maximum wing-rock amplitude. Figure 7 provides the (α, θ_A) boundary for starting wing rock. To determine the maximum possible oscillation amplitude, one needs to repeat the computations,[11] producing Fig. 5 for apex half-angles θ_A different from 10 deg.

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Correlation for the Estimation of Afterbody Drag with Hot Jet Exhaust

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Introduction

MODERN turbojet and turbofan engines of combat aircraft operating over a wide range of power settings experience jet exhaust temperature typically varying from 1000–2000 K, whereas much of afterbody-nozzle testing is conducted with a cold jet near 300 K.[1–5] Thus there remains a problem to determine the extent to which jet total temperature (and its associated gas constants) affects the afterbody drag of a combat aircraft under various operating conditions of its nozzle during the flight operation.[6–8] Physical modeling of jet freestream interactions with temperature effects is quite difficult; and, calculations of afterbody drag with hot jet exhaust are computationally intensive. Efforts made earlier for the es-

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